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Mr. DUNKIN, F.R.S., Vice-President, in the Chair.

Lieut. T. Preston Battersby, R.A., Cromlyn, Rathowen, Westmeath, Ireland;

was balloted for and duly elected a Fellow of the Society.

On Mean Refractions. By E. J. Stone, M.A. F.R.S.

In the *Monthly Notices* for December 1867 there is a paper of mine on Bessel's Mean Refractions. The results there given were obtained from a discussion of the circumpolar observations made at Greenwich from 1857 to 1865. These observations were divided into five groups with N.P.D. $0°-15°$; $15°-30°$; $37°-37°\ 40'$; $37°\ 40'-43°$; $43°-46°\ 29'$; and, when the observations were reduced with Bessel's Refractions, the following values of excess of N.P.D. below pole over N.P.D. above pole were found for the five groups :—

	(1)	(2)	(3)	(4)	(5)
	"	"	"	"	"
A	+0·240	+0·196	+0·570	+1·054	+2·190.

The theoretical weights are

130	88	31	54	28.

Then assuming that the true co-latitude

$$= 38°\ 31'\ 21''{\cdot}80 + \tfrac{1}{2}x,$$

and True Refraction = Bessel's Refraction $(1-y)$, a notation which will be adopted throughout the present paper, the residuals A are reduced to the following, when $x = -0''{\cdot}41$ and $y = 0{\cdot}0053$,

	(1)	(2)	(3)	(4)	(5)
		"	"	"	"
B	+0·15	−0·10	−0·14	−0·08	+0·10

The residuals A are out of all proportion to the theoretical probable errors of the groups; those of B are certainly well within them. The results of this paper were at the time accepted by the Astronomer Royal and used in the reduction of the observations 1868 to 1876.

It is, however, no great surprise to me to find, that the Astronomer Royal now proposes to abandon the use of these diminished Refractions and to revert to Bessel's unaltered Refractions. As a matter of convenience, pending further enquiry, I believe such a step may be expedient. But, while fully admitting such to be the case, I must call attention to the very slight evidence upon which the rejection of these diminished Refractions is based, and to some points of an unsatisfactory character in Bessel's Refractions.

When it was proposed, in 1868, to use the diminished Refractions it was, of course, known that the N.P.D.'s of the stars observed at Greenwich would systematically differ from the corresponding observations contained in the Catalogues for 1860 and 1864; such discordances do not show whether the old refractions or the new refractions are correct; they merely show that they are different.

With respect to the Sun observations the following are the values of z which were found 1857–1869 in the investigations of the positions of the plane of the ecliptic.

Year.	z.	
	"	
1857	−0·640	
1858	−0·679	
1859	−0·683	
1860	−0·438	
1861	−0·600	Bessel's Refractions were used, and the Transit-Circle cube was unpierced, until 1865, August.
1862	−0·646	
1863	−0·689	
1864	−0·983	
1865	−0·510	
Mean	−0·641	
1866	−0·004	Bessel's Refractions were used, but the Transit-Circle cube was pierced.
1867	+0·118	
Mean	+0·057	
1868	+0·757	
1869	+0·487	
Mean	+0·622	Bessel's Refraction (1 − 0·0053).

The greatest change which could be introduced into the value of z by the change of refraction does not exceed $+0''\cdot7$. So far, therefore, as the observations 1857–1865 are concerned, these Sun observations do strongly confirm the apparent necessity of diminishing the refractions by $0\cdot0053 \times R$. But it is clear that the piercing of the cube, or the use of the new collimators, or some other instrumental changes introduced at the time of the piercing of the cube, have largely affected the values of z, and, before we can appeal to these results in proof of the accuracy of Bessel's Refractions, or of the refractions diminished in the proportion 0·9947 : 1, we must settle which of the two series of observations is the freer from systematic error.

And with respect to the comparison which is given in the Addendum to the Nine Year Catalogue between the results of that Catalogue and the Melbourne Catalogue of 1227 Stars for 1870, I must remark that the large systematic corrections which have been applied to the Melbourne results for supposed errors in Bessel's Refractions should be removed before this agreement is appealed to as a proof of the accuracy of Bessel's Refractions.

To show the importance of this point, I give the results of a comparison between the corrected Greenwich Catalogue before and after the corrections which have been applied to the Melbourne Catalogue have been removed.

The corrections applied to the Melbourne North Polar Distances were obtained from a discussion in my paper of 1867, December, of the Melbourne observations 1863, 1864, 1865. These observations were compared with the results of the Greenwich Catalogue 1860, and the corrections were deduced by forcing a close agreement between the results. It is therefore rather arguing in a circle to appeal to an agreement between the corrected Melbourne results and those of the Nine Year Catalogue, also corrected and made to agree with the Catalogue of 1860, as a proof of the accuracy of the corrections applied to the Greenwich Catalogue of 1872 or as a proof of the accuracy of Bessel's Refractions.

N.P.D.	M 1 Melbourne, with Stone's corrn. − Nine Year Cat. corrd. to 1860.	M 2 Melbourne, uncorrd. − Nine Year Cat. corrd. to 1860
°	″	″
48	+0·38	−2·32
52	+0·26	−1·66
56	+0·05	−1·65
60	−0·20	−1·35
64	−0·47	−1·40
68	−0·65	−1·41
72	−0·70	−1·33
76	−0·53	−1·06

N.P.D.	M 1 Melbourne, with Stone's corrn. } − (Nine Year Cat. corrd. to 1860.)	M 2 Melbourne, uncorrd. } − (Nine Year Cat. corrd. to 1860.)
°	″	″
80	−0·22	−0·66
84	+0·02	−0·34
88	−0·10	−0·39
92	−0·09	−0·33
96	−0·17	−0·35
100	−0·22	−0·35
104	−0·26	−0·35
108	−0·16	−0·20
112	−0·15	−0·15
116	−0·06	−0·02
120	+0·09	+0·17
122	+0·44	(+0·53)
124	−0·47	(−0·36)

The last two residuals contain observations which have been reduced with Bessel's Refractions diminished by 0·0023 × Tabular Refraction and 0·0049 × Tab. Refraction respectively, and are therefore enclosed in brackets.

The agreement M1 may well have been considered satisfactory; but it will be seen from the system M2 that Bessel's Refractions must be diminished to represent either the Melbourne or the Greenwich observations.

With respect to the Cape Catalogue of 1159 Stars for 1860, a comparison was made by me with the results of the Greenwich Catalogue for 1860, before the Cape Catalogue was committed to the press, and it was chiefly because I saw that there was a rather close agreement between its results and those of the Greenwich Catalogue that I adopted Bessel's Mean Refractions unaltered instead of attempting to apply any corrections to my results. It will be seen on reference to the Cape Catalogue, pages 9 and 10, that, from a discussion of the circumpolar observations, I obtained a diminution of Bessel's Refractions amounting to 0·0047 × Tabular Refraction; but on account of the Zenith Distance of the Pole at the Cape being more than 56°, it is very difficult to separate the errors of the Refraction tables adopted from the error of assumed co-latitude, and I thought, therefore, pending further enquiry, it would be better to use Bessel's Refraction Tables unaltered than to apply an uncertain correction.

The recent Cape observations do appear to offer grounds for serious doubts about the necessity of decreasing Bessel's Refractions, for small zenith distances, by 0·0053; but the evidence thus collected is not certainly such as should outweigh the accumulated evidence of the Greenwich circumpolar observations

unless it can be shown that the residual errors presented by the Greenwich results can be otherwise accounted for.

And it appears to me that, if any serious doubts have ever been entertained on the substantial accuracy of my work of 1867, great advantages would have resulted had the results been independently recomputed, and, whether they confirmed or disproved the accuracy of my work, printed in the Addendum of the Greenwich Nine Year Catalogue. If my work could have been shown to have been seriously inaccurate, there would have been an end of all difficulty, so far as the evidence there collected was concerned. On the other hand, if my work was confirmed and the residual errors, which I have given, found to be substantially correct, then, most certainly, in the best interests of the Greenwich Observatory, these residuals should be kept distinctly in view and not virtually shelved from false impressions of the inaccuracy of my work. For, so far as I can see, there are only four possible causes to which these residual errors can be referred:—

(1.) My work is wrong—*i.e.* the residual errors which I have given are not correctly deduced from the observations, and do not, therefore, represent their results as given in the annual volumes of the Greenwich Observatory.

(2.) The Greenwich observations, as reduced and published in the annual volumes of the Greenwich Observatory, are affected with systematic errors sufficiently large to give rise to the residual errors contained in the five groups A.

(3.) Bessel's Refractions do require to be diminished by about 0·005 × Bessel's Refraction; and the co-latitude of Greenwich does not differ much from 38° 31′ 21″·60.

(4.) Bessel's Mean Refractions have been deduced so imperfectly from theory as to give rise to these residual errors.

The residual errors A may, of course, arise, not from one, but from the conjoint action of two or more of these causes of error.

First, with respect to the accuracy of my work. The only evidence, so far as I am aware, which has been brought forward to show that my results are not reliable are the facts that the thermometer used during the year 1857 and part of the year 1858 read too high by about six tenths of a degree Fahrenheit, and that the barometer used read too low by about a hundredth of an inch. It has also been assumed that the necessary corrections had not been applied by me. I need hardly point out that these instrumental errors, if unallowed for, would not have affected my determination of y by more than 0·0016 had the thermometer with a large index error been used during the nine years over which my discussion extends, instead of only during the year 1857 and a part of the year 1858. Besides this, the thermometer read too high, and the Barometer too low, and the Refractions, therefore, used in the Greenwich reductions were already smaller than they would have been if correctly deduced from Bessel's

Tables. I ought, therefore, so far as these causes were effective, to have obtained an increase and not, as I did, a decrease of the Refractions used. At the January meeting of the Society, Mr. Christie did bring forward some results derived from the observations 1857 to 1865 which I, at first, hoped might have settled the simple point whether the residual errors A had been correctly deduced from the results of the Greenwich volumes 1857 to 1865. The observations were divided by Mr. Christie into five groups like mine, and were given on two assumptions: first, that the law of discordance R—D was $= a$ sin Z.D.; and secondly, that it was $= a$ sin Z.D. $\cos^2$ Z.D. The mean errors for the five groups were as follows:—

(1)	(2)	(3)	(4)	(5)	R–D
+0·19	−0·09	+0·22	+0·79	+1·20	a sin Z.D.
+0·21	+0·09	+0·49	+1·11	+1·56	a sin Z.D. $\cos^2$ Z.D.

whilst my results obtained in 1867 are

+0·24	+0·20	+0·57	+1·05	+2·19.

There is a rough agreement amongst all these values, and they all show that Bessel's Refractions unaltered do not represent the Greenwich observations; but the differences between the results given by Mr. Christie and myself differ with regard to the most important residual (5) by more than four times the theoretical probable error of my result, and either Mr. Christie's results or mine are in error, for the differences of the (R—D) corrections will not account for the discordances.

In my changes of residence between England and the Cape, I have, unfortunately, lost or mislaid the calculations upon which the results given in the paper of December 1867 were based. It is impossible for me to assert positively, from memory, what small corrections I applied to the results more than thirteen years ago; but from a general knowledge of the interest I took at the time in the investigation, and from my general rule of work, I have felt throughout that I must have applied all the corrections which could be applied with certainty to the results; but as these questions of inaccuracy have been raised, I have now recomputed the principal residual for group (5).

It may probably avoid useless discussion if I state clearly all the corrections applied to the Greenwich results.

First:—The observations have been extracted from the annual Catalogues of the Greenwich Observatory 1857 to 1865 inclusive.

Secondly:—I have not used the Reflexion observations. These observations have been already combined with the direct observations in the determination of the Nadir Points and in the deduction of the R—D corrections, and the corrections thus deduced have been applied in the Greenwich volumes to the

direct results. With this exception, all the results given in the Greenwich volumes have been used.

Thirdly:—I have assumed that the thermometer used in 1857 and part of 1858 read too high by six tenths of a degree Fahrenheit.

Fourthly:—I have assumed that the barometer used in the reductions read too low by a hundredth of an inch.

Fifthly:—As some of the observations contained in the group (5) have a greater zenith distance than 82°, and these observations have had their Refractions already decreased by 0·0023 × Tabular Refraction, I have therefore increased their Refractions by the same amount to bring them back to the results which would have been obtained had Bessel's Refractions been used unaltered.

Sixthly:—The observations have all been reduced to an assumed co-latitude 38° 31′ 21″·80.

Seventhly:—I have weighted the observations from the formula

$$W = \frac{1}{2\epsilon^2 + \frac{e^2}{n} + \frac{e_1^2}{n_1}}$$

assuming $2\epsilon^2 = 0{\cdot}04$, and extracting e and e_1 from my paper on probable errors in the *Monthly Notices* for 1869. The result obtained in 1867 was

2″·19 with a weight 28 (or probable error ±0″·2).

The result which I now obtain is

2″·10 with a weight 24 (or probable error ±0″·2).

The two results are essentially identical, and it is clear that all the corrections now applied were applied in 1867, but (as my paper on probable errors was not written before 1869) I must have worked out, with sufficient approximation, probable errors at different zenith distances and used them in my work: I believe, therefore, that I may safely assert that my work of 1867 correctly represents the results of the Greenwich observations as printed, and that we must look to one of the following causes for the solution of any difficulties presented by the appearance of the residuals A.

I now come to the consideration of the assumption (2).

There are undoubtedly systematic errors in the Greenwich results; but not such as can account for the residuals A.

During the years 1857 to 1865 there was no well-marked constant difference between the Nadir Point determinations with the reflecting eyepiece and from stars, and the differences between the Nadir Points from Northern and Southern stars were confined within small limits.

And not only is this the case but, so far as I know, no good reasons can be assigned why the small uncorrected systematic

instrumental errors should tend to increase rather than decrease the residual A (5). The determination of the horizontal flexure and its correction under the form a sin Z.D. should prevent any serious instrumental errors affecting the circle readings near the horizon; and although the good effects of this were virtually destroyed by the system formerly adopted at Greenwich of introducing another correction of the form b sin Z.D. to destroy the the mean difference between the Nadir Points determined from Northern and Southern stars, yet the corrections thus introduced, during the period 1857 to 1865, were not large, and have not affected the residual error (5) by $0''.3$. The law assumed, since 1862, to destroy the difference between the results for Northern and Southern stars has been a sin Z.D. $\cos^2$ Z.D., and a term under this form must exist in the flexure unless the yielding of the instrument in the two horizontal positions object-glass North and South remains unchanged, a fact which can only be known from experiment, and appears to be disproved when systematic differences are found to exist between the Nadir determinations from the Northern and Southern stars. So far as the observations 1857 to 1865 are concerned I can see no reason whatever for assuming it possible that the residual error A (5) has been increased $0''.5$ from uncorrected systematic errors.

I now pass to the consideration of the assumption (3).

Upon this point I appeal to a comparison of the residual errors A and B; and, as a confirmation of this result, I give the corresponding residual errors from the Greenwich observations 1868–1876.

The residuals for the first three of the following groups are extracted from the Introduction to the Nine Year Catalogue, 1872. The two last are not given in that Introduction. But, as these are all important in questions of Refraction, I have extracted the necessary observations from the Introductions to the annual volumes of Greenwich Observations 1868 to 1876, and formed the mean errors with the proper theoretical weights.

1868–1876.

Excess of N.P.D. below over N.P.D. above pole.

(1)	(2)	(3)	(4)	(5)
$''$	$''$	$''$	$''$	$''$
−0·26	−0·63	−0·36	−0·72	−0·19

The first three groups give a correction of $-0''.41$ to the assumed co-latitudes, and, after applying this, the residuals become

	(1)	(2)	(3)	(4)	(5)
	$''$	$''$	$''$	$''$	$''$
C	+0·15	−0·22	+0·05	−0·31	+0·22

These residual errors are small, and certainly do not afford the slightest indication of any serious errors in the adopted refractions, which are those of Bessel diminished by 0·0053 × Tabular Refraction. The evidence afforded by a comparison of the

residual errors A and B and the smallness of the residual errors C, where the groups (4) and (5) contain observations extending to 85° and below 85° Z.D. respectively, appear to afford evidence in favour of the reduced refractions which it must be difficult to resist, unless it can be shown that these results can otherwise be accounted for. I believe, however, it will be proved, in the present paper, that it is possible to account for these results, to a great extent, at all events, under the Section (4).

This point appears important, and will probably meet with the more ready acceptance, as it does not necessitate any considerable changes in Bessel's refraction at the smaller zenith distances, and accounts for the different values of y which have been obtained from discussions extending over small and large ranges of zenith distance.

The mean refractions of the *Tabulæ Regiomontanæ* are merely those of the *Fundamenta* multiplied by 1·003282. But at the zenith distance 85° these refractions were found by Bessel to be too large, and he recommended that they should be replaced by quantities which he had derived more directly from observation. Attempts have been made from time to time to deduce corrections to Bessel's refractions by assuming, as Bessel had done, that the tabular refractions required some constant multiplier or divisor. These attempts have led to anomalous results. If we start with Bessel's Refractions—then, as might be expected, no corrections are found necessary for small zenith distances; at considerable zenith distances the tabular refractions are found to be too large, and the factor by which they require diminution increases with the zenith distance. The whole theory of Bessel's mean refractions is included in the following relations:—

$$(1)\quad \frac{d\mathrm{R}}{d\mu} = \frac{1}{\mu}\,\frac{\mu_0 \sin z}{\sqrt{(1+x)^2\mu^2 - \mu_0^2 \sin^2 z}} = \phi'(\mu) \text{ suppose.}$$

$$(2)\quad \mu^2 = 1 + 4k\rho,$$

$$(3)\quad \rho = \rho_0 \epsilon^{-\beta x},$$

where R is the refraction, μ, ρ the index of refraction and density of the air at a height $x \times$ Earth's radius; μ_0, ρ_0 the corresponding quantities at the Earth's surface; κ a constant; β a quantity introduced to force an agreement between the calculated and observed refractions at considerable zenith distances; ostensibly β is introduced to give the law of diminution of ρ with increasing altitudes, but its real object is that which I have indicated. It is clear that the relation $\rho = \rho_0 \epsilon^{-\beta x}$ is not the true relation which exists between the quantities ρ, ρ_0 and x. The absolute truth of this formula would require that the pressure and temperature of the air should be uniform over the Earth's surface. It is, I believe, equally clear that ρ cannot be expressed as a simple function of ρ_0 and x. All such functional relations must tacitly assume that the atmosphere is in equilibrium, thermal and otherwise, whereas we know that the changes are largely, perhaps principally, due to diffusion.

But whatever may be thought of the theory from which Bessel's Refractions have been deduced, they certainly represent the observations with considerable accuracy. If β be determined with reference to the horizontal refraction the errors of the computed refractions are still not very great. And if we determine β so that the calculated refraction agrees with observations at Z.D., 89°, 88°, 87°, 86°, and 85° respectively, we are certain to obtain theoretical curves, in succession, which lie closer and closer to the true curve of refraction. If, therefore, we could insure the near coincidence of the curves for the smaller zenith distances by the proper determination of ρ_0 or its equivalent μ_0, we should, with such a theory as Bessel's, be able to obtain any necessary degree of accuracy over a considerable range of zenith distance. It will be seen that I regard the theory chiefly as a useful guide to a convenient formula of interpolation. Since the mean refractions of the *Tabulæ Regiomontanæ* have not been directly computed from the formulæ, but derived from those of the *Fundamenta* by the mere multiplication of these refractions by a constant quantity, we can only indirectly infer the values of of μ_0 and β which have been really assumed by Bessel.

It appears that Bessel's refractions at Barometer 30 inches and Thermometer 50° can be sensibly reproduced from the theory when

$$\mu_0 = 1{\cdot}00028272, \log. \beta = 2{\cdot}8699628.$$

The refractions thus computed were found by Bessel too large at 85° Z.D., and I believe that all subsequent investigations have led to a similar result. There may be, and probably will be, some doubt about the exact excess of Bessel's Tables at 85° Z.D., but I shall assume that the true refraction for the Barometer 30 inches and Thermometer 50° is $589''{\cdot}13$.

Any change in the adopted value of the refraction at 85° will, of course, introduce corresponding changes into the results for the other zenith distances, but the necessary changes can be inferred from the differences between my results and those given by Bessel.

When Bessel found the refractions of the *Fundamenta* multiplied by 1·003282, although agreeing well with observations at the smaller zenith distances, too large by about $2''$ at the zenith distance 85°, he should, in my opinion, have recomputed his refractions and determined the quantity β so that the necessary agreement near 85° Z.D. was obtained. This is what I have done. And, if the value of the refraction at 85° which I have assumed be nearer the truth than that deduced from Bessel's Tables, then the intermediate refractions given in my table must also be closer to the true results than Bessel's are.

The following table exhibits the values of the refraction for Barometer 30 inches and Thermometer 50°, which I have calculated, and the corresponding quantities computed from Bessel's *Tabulæ Regiomontanæ*.

I adopt

$$\mu_0 = 1{\cdot}0028272 \text{ and } \log.\ \beta = 2{\cdot}8505582.$$

Refractions for Barometer 30 *in. and Thermometer* 50° *F.*

Z.D.	Stone's Refraction.	Bessel's Refraction.	Excess of Bessel.
° ′	″	″	″
45	58·24	58·22	−0·02
55	83·08	83·06	−0·02
65	124·33	124·35	0·02
70	158·72	158·76	0·04
75	213·96	214·10	0·14
76	229·39	229·55	0·16
77	246·97	247·18	0·21
78	267·24	267·52	0·28
79	290·83	291·22	0·38
80	318·70	319·16	0·46
81	351·93	352·58	0·65
82	392·39	393·32	0·93
83	442·57	443·85	1·28
83 30	472·35	473·95	1·60
84	506·15	508·06	1·91
84 30	544·74	546·98	2·24
85	589·13	592·00	2·87

There appears a slight, but not important, difference between my results and Bessel's at 84° 30′. I believe my results are correct, but the calculations are intricate, and I may have made some slight error.

The importance of the changes thus introduced into Bessel's Refractions will, perhaps, be more apparent if we consider their effects when we attempt to compare Bessel's theory with observations by discussing circumpolar observations under the assumption that the true refraction is equal to Bessel's Refraction × $(1-y)$, and consider y to be constant over any wide range of zenith distances.

If $38° + \frac{1}{2}x$ be the assumed co-latitude and we have observations above and below pole at zenith distances 38°, 38°; 75°, 1°; 82°, −6°, 84°, −8°, our equation of condition will be of the form

$$(1)\quad x + 90y = a_1,$$
$$(2)\quad x + 215y = a_2,$$
$$(3)\quad x + 387y = a_3,$$
$$(4)\quad x + 500y = a_4,$$

when a_1, a_2, a_3, a_4, are the excess of observed N.P.D. below pole over that observed above pole reduced with Bessel's refractions.

If, therefore, Bessel's Refractions are in error by the differences between my computed quantities and those given in Bessel's Tables—and this must be the case if Bessel's ρ_0 be adopted and the assumed error of his tables at 85° Z.D. be $2''{\cdot}87$—then, when the observations are sufficiently numerous and good to destroy any mere errors of observation, our equations of condition would assume the form

$$\begin{aligned}
(1)\quad & x + 90y = 0''{\cdot}00,\\
(2)\quad & x + 215y = +0''{\cdot}14,\\
(3)\quad & x + 387y = +0''{\cdot}93,\\
(4)\quad & x + 500y = +1''{\cdot}91,
\end{aligned}$$

from which, for the determination of x and y, we have

$$\begin{aligned}
4x + 1192y &= +2''{\cdot}98,\\
1192x + 454094y &= 134''{\cdot}501,
\end{aligned}$$

which give

$$x = -0''{\cdot}626, \quad y = +0''{\cdot}0046.$$

The residual errors would be the following :—

$$\begin{aligned}
(1)\quad & -0''{\cdot}212,\\
(2)\quad & +0''{\cdot}223,\\
(3)\quad & +0''{\cdot}224,\\
(4)\quad & -0''{\cdot}236.
\end{aligned}$$

A comparison should be made between these results and those obtained in 1867.

The residual errors are not large ; and for the groups (3) and (4) are certainly not larger than the residual errors would be had they been derived from a large number of good observations.

It appears, therefore, that when the two quantities ρ_0 and β are both determined indirectly from a discussion of observations, we have in Bessel's Theory an *embarras de richesses*. If we assume that Bessel's Refractions in the *Tabulæ Regiomontanæ* are too great at 85° by $2''{\cdot}87$, then we can force a somewhat close agreement between our circumpolar observations and Bessel's Refractions, either by adjusting the refractions by a redetermination of β, or by reducing all the refractions in the proportion of about 0·996 to 1, which agrees nearly with the result obtained in my paper of 1867, December. In the first case the Refractions at small zenith distances will remain unaltered, and, if our co-latitude has been originally deduced by the use of Bessel's Refractions, our co-latitude will also remain unaltered. In the second case all the Refractions will be proportionately altered and the assumed co-latitudes require correction of about $0''{\cdot}3$. If we restrict the arc over which our discussion extends, we obtain smaller and smaller corrections to the assumed values of our Refractions and co-latitude, and by sufficiently contracting the range of zenith distances we shall merely reproduce our original assumptions.

Our real difficulty in these discussions appears to be to determine, amongst the small quantities involved in the equations of condition as residual errors, what proportion of the adjustment should be made with μ_o or ρ_o and what with β.

It will be seen that the results which I have obtained explain those which Mr. Main deduced from a discussion of the observations below Z.D. 70° made during the years 1836 to 1854.

At 82° my Refraction is less than Bessel's in the proportion of 0·9976 to 1, and at 85° in the proportion of 0·9949 to 1. These results agree almost identically at 82° and 85° with those obtained by Mr. Main, but it was formerly assumed that no sensible corrections were needed at smaller zenith distances than 82°, and that the correction 0·9976 : 1 continued sensibly unchanged from 82° to 85°. In my calculations the changes are, as they must be, progressive; but of course there may be some doubt whether my corrections at 85° may not be slightly too large.

On account of the rapidity with which the differences between the refractions given in the present paper and Bessel's Refractions increase at considerable zenith distances it is very difficult to correct the mean results when observations are combined with zenith distances differing by several degrees. I have, therefore, given the results for the different stars from 82° to 85° Z.D., 1857 to 1865, and also for stars 80° to 85° during the years 1868 to 1876. It may be noticed that the adopted co-latitudes for the same Refractions are different in the two sets of observations. The co-latitudes adopted are those given by the groups least affected by refraction. The co-latitude during the second period appears to have been affected by the constant difference which existed during the period 1870–1874 between the Nadir Point determinations, and the virtual rejection of the observations with the reflecting eyepiece, by the application of constant corrections to make their results agree, in mean results, with the zenith points found from the star observations. To some extent, however, any error which might arise from the course adopted will be compensated for when the co-latitude obtained from the observations themselves is adopted, as in the present comparison.

Observations 1857–1868.

Star's Name.	App. Z.D. at S.P.	Weight.	Excess of N.P.D. below, over N.P.D. above, Pole. Bessel's Ref. Co-latitude	R = Bess. (1 − 0·0053) Co-latitude	Ref. of present paper. Co-latitude
	° ′		38°31′21″·80	38°31′21″·60	38°31′21″·92
Capella	82 40	5·5	+1·34	−0·30	+0·07
δ Cygni	83 41	4·7	+2·81	−0·80	+1·04
ω Ursæ Maj.	84 37	0·9	+3·50	+1·24	+1·08
13 Lyræ	84 45	2·3	+3·86	+1·46	+1·11
β Aurigæ	83 36	1.2	+3·07	+1·07	+1·18
α Cygni	83 44	0·6	+4·49	+2·49	+2·60
11 Lacertæ	84 57	0·8	+1·34	−1·17	+1·65

The co-latitude 38°31′21″·92 has been obtained from group (1) A, and the residual errors for Bessel's Tables should be referred to the same co-latitude as the refractions are equal for the smaller zenith distances.

The mean results will then be :—

With Bessel's Refractions.	Tab. Ref. (1 − 0·0053).	Refrs. of the present Paper.
″	″	″
+2·26	+0·39	+0·65

Observations 1868–1876. Co-latitude, 38°31′21″·60.

	°	′				
χ Ursæ Maj.	80	2	1.81	+1·51	−0·14	+1·06
ψ Ursæ Maj.	83	19	1·62	+1·51	−0·14	+1·06
α Cygni	83	44	3·02	+2·33	−0·19	+0·73
δ Persei	81	8	0·88	−0·98	−2·81	−1·58
θ Persei	79	50	0·58	+1·41	−0·25	+0·95
13 Lyræ	84	45	0·28	−3·52	−6·49	−6·03
δ Cygni	83	41	2·74	+2·29	+0·26	+0·76
ω Cygni	79	33	1·67	−0·47	−2·09	−0·89
τ Herculis	81	52	0·99	+0·57	−1·42	−0·22
ξ Cygni	85	10	0·48	+0·76	−2·34	−2·08
ι Ursæ Maj.	79	57	6·18	+1·70	+0·04	+1·24
f Cygni	81	32	4·43	+1·37	−0·60	+0·58
Capella	82	40	9·00	+0·87	−1·29	−0·16
β Aurigæ	83	36	4·00	+2·01	−0·47	+0·46

The mean values are

″	″	″
+1·33	−0·71	+0·35,

but from the co-latitude investigation the second group requires a correction +0″·40, and the first and third, on account of differences of adopted refraction, a correction of +0″·40 − 50 or −0″·10.

The mean errors then become

″	″	″
+1·23	−0·31	+0·25.

Both comparisons show that Bessel's uncorrected results give much larger residual errors than those obtained either by diminishing these refractions by 0·0053 or using the refractions of the present paper.

These results have had several corrections applied to them, and do not profess a greater accuracy than 0″·1. The errors may in particular cases amount to 0″·2 ; but I think this will be the extreme range of error.

The systematic errors of the Greenwich Results during the second period are larger than during the period 1857–1868, but are still confined within limits of less than a second of arc.

So far as these observations can afford us information it appears that I have not over-corrected Bessel's Refractions at 85°.

The chief defect in Bessel's Mean Refractions, which appears to have been clearly proved, is the running away of his Refractions at considerable zenith distances. It appears from the results of the present paper that this can be corrected to 85° Z.D. by a proper determination of β without the necessity of any sensible alteration of the Refractions at small zenith distances. It would appear, therefore, that if we can feel confidence in Bessel's value of ρ_0 or μ_0, or in his Refractions at small zenith distances, his Tables must be considered a satisfactory practical solution of the problem of astronomical refraction for the present epoch. But I cannot feel that perfect confidence in Bessel's results which astronomers in general appear to feel. The tables in use have been obtained by a comparison between Observations and the Tables of the *Fundamenta*, on the supposition that Refraction = Tab. R. $(1-y)$; and the constant thus found, $(1-y) = 1{\cdot}003282$, must depend upon the range of Zenith distances over which the discussion extended. But the result appears to disprove the assumed constancy of y, for the Refractions at 85° are too large, and can only be adjusted to the Refractions at the smaller zenith distances by applying corrections which no longer render y a constant even over an arc of 75° Z.D.

And when we are restricted to a range of zenith distances of 75° the difficulties of determining y within such limits as 0·003 appear to me very great indeed. How great these difficulties are may best be seen by a numerical illustration. If we attempt the solution of this problem by a discussion of circumpolar observations in latitude 52° our limiting equations of condition will be about

$$x + 90y = a_1, \qquad x + 215y = a_2,$$

and an error of 0·003 in y cannot introduce into our residuals a_1 and a_2 greater differences than 0″·38, and it is from these differences alone that the value of y has to be deduced. Nor are we in a better position if we attempt to determine y from a discussion of the results at a Northern and Southern Observatory—Greenwich and the Cape, for instance; our limiting equations will then be confined within some such limits as

$$x + 116y = a_1, \qquad x + 230y = a_2,$$

and the extreme differences introduced in the residuals a_1 and a_2 by changes of 0·003 in y will not amount to more than 0″·34. When, therefore, it is considered that at 75° Z.D. the Refractions are already in doubt by 0″·14, on account of the corrections required by β, and that the results of our best Circles are not free from small systematic errors, it will, I think, be admitted that it is very

difficult, if not hopeless, to expect to determine y from such equations within such errors as 0·003. Whatever may be the difficulties connected with the making of good observations at considerable zenith distances, and in obtaining a theory of Refraction sufficiently strong to brace together the results with those made at smaller zenith distances, I believe it is absolutely necessary that we should extend our enquiry to sufficient zenith distances to separate the errors in the adopted refraction tables from the systematic instrumental errors to which our Circle observations are liable, and I cannot regard this as done when discussions are limited to zenith distances less than 75°. But these general considerations are not the only grounds upon which I doubt the extreme accuracy of Bessel's determination of μ_0.

It will be found a fundamental assumption of his work that at the limits of the atmosphere the density is so small that with his assumed law $\rho = \rho_0 \epsilon^{-\beta x}$ the integration may be extended to infinity without sensibly affecting the resulting refractions. But in this case we have sensibly, at the limits of the atmosphere, $\mu = 1$; and returning to our equations,

$$\frac{dR}{d\mu} = \frac{\mu_0}{\mu} \frac{\sin z}{\sqrt{(1+s)^2\mu^2 - \mu_0^2 \sin z}} = \phi'(\mu),$$

we have

$$R = \int_1^{\mu_0} \phi'(\mu) d\mu = \phi(\mu_0) - \phi(1)$$

$$= \phi'(\mu_0) \cdot (\mu_0 - 1) - \phi''(\mu_0) \cdot \frac{(\mu_0-1)^2}{1 \cdot 2} + \ldots.$$

The first term, $\frac{1}{\mu^0}(\mu_0 - 1) \tan Z$, is independent of the law of variation between s and μ, and is, for small zenith distances, the only important term. In fact, with Bessel's Theory the remaining terms do not amount at 45° Z.D. to 0″·1.

If we accept Bessel's value of μ_0, we find for the first term, in seconds of arc, 58″·29. But μ_0 is supposed to be the refractive index of air at barometer 30 inches and thermometer 50°F. The value of μ_0 for dry air has been given by Biot $= \sqrt{1{\cdot}000588768}$ at barometer 0^m·76 and thermometer 0° C.; and this would correspond to the value $\mu_0 = 1{\cdot}00028462$ at 30 in. and 50°F.; but this value of μ_0 gives 58″·69 for the coefficient of tan Z. The difficulty presented by these differences has been, I believe, generally explained by the fact that Biot's value is for dry air, whilst Bessel's is for the moist air of the atmosphere. It appears to me that this explanation cannot be the true one. The coefficient of refraction should be larger for the moist air instead of smaller.

I have taken the trouble to compute the refractions which would result if we adopted Biot's value of μ_0 and Bessel's Theory.

The refractions thus deduced do not appear to me admissible. If Biot's determination of μ_0 for dry air is too large, this should be proved by an independent determination of that quantity; but

accepting its value we must, I fear, abandon the assumption that $\mu = 1$ at the limit of the Earth's atmosphere, and assume that its value is about 1·0000019.

In this way we can make Biot's value of μ_0 give us Bessel's Refraction at Z.D. 45°, for

$$R = \int_{\mu_0'}^{\mu_0} \phi'(\mu) \, . \, d\mu = \phi'(\mu_0) \, . \, \frac{\mu_0 - \mu_0'}{\mu_0} - \ldots .$$

and the first term $= \tan z \, . \, \frac{\mu_0 - \mu_0'}{\mu_0} = 58''\cdot29$ as before.

This supposition gives with Bessel's law of density a height of about 27 miles to the atmosphere.

But, if this explanation be accepted, the agreement which is found between Bessel's Refractions and the observations at the smaller zenith distances is a forced agreement, arising from the errors of the theory being counteracted by an erroneous determination of μ_0, and although the errors thus introduced at considerable zenith distances may be destroyed by the aid of the second constant β, we must be liable to a separation between the true refractions and those obtained from such a strained theory at intermediate and perhaps unexpected points; such errors, if they were small, as they must be, would scarcely be found without a systematic examination, with a considerable number of observations, of the results at many of the intermediate zenith distances.

Upon this work, and an attempted revision of Bessel's Theory, on the lines here indicated, I am at present engaged.

A List of 410 *Radiation-points of Shooting Stars, deduced from Observations in Hungary in the Years* 1871 *to* 1878, *and of* 80 *probable Radiants deduced from the* 410 *Radiation-points.* By Prof. N. de Konkoly, Member of the Hungarian Academy at Budapest, Director of the Observatory at O'Gyalla.

The present paper is based upon observations in Hungary, in the years 1871 to 1878, of 2999 shooting stars; but of these only 1641 are used for the determination of Radiation-points, the others being rejected as relating to sporadic stars, or as being bad observations.

I have deduced from this large number of observations 410 Radiation-points.

The shooting stars were observed in Hungary at 5 stations:—

	Long. h m s	Lat. ° ′ ″
1 Astrophysical Observatory, O'Gyalla	λ=0 7 13·9 E.*	D=47 52 43·4
2. Real School at Agram	λ=0 1 30·7 W.	D=45 50
3. Forest Academy, Schemnitz ...	λ=0 10 5 E.	D=48 27
4. Economical School at Hodmezövasárhely	λ=0 15 1 E.	D=46 26
5. Gymnasium at Szathmárnémethi	λ=0 26 1 E.	D=47 46

The observers were : at O'Gyalla, the staff of the Observatory ; at Agram, Professor Stozir; at Schemnitz, Professor Dr. Schwarz; at Hodmezövásárhely, Professor Nagy, and at Szathmárnémethi, Professor Toth.

I arrange as follows the whole series of observations, to show how they are distributed in the 8 years.

The first column gives the year; the second, the number of days on which observations were employed for the determination of Radiation-points; the third shows the number of Radiation-points ; and the fourth shows how many shooting stars were employed for the calculation of the Radiation-points. I give separately the results for each station, and finally the sum from all the stations.

Table of Distribution of Meteors in different Years.

O'Gyalla—Observatory.

Year.	Observing Days.	Number of Radiation-points.	Number of Meteors employed.
1871	3	13	44
1872	12	71	278
1873	10	33	105
1874	7	68	271
1875	10	84	361
1876	6	22	89
1877	3	7	28
1878	8	18	66
Sum	59	316	1242

Schemnitz—Academy.

1875	4	8	36
1876	2	6	20
1878	2	3	10
Sum	8	17	66

* E. or W. from Vienna Observatory.

Year.	Observing Days.	Number of Radiation-points.	Number of Meteors employed.
	Hodmezövásárhely—Economical school.		
1875	Only two sporadic fireballs.		
1876	5	29	120
1877	1	2	6
1878	3	8	31
Sum	9	39	157
	Agram—R. Real School.		
1875	2	11	37
1876	2	4	12
1877	3	10	52
Sum	7	25	101
	Szathmárnémethi—Gymnasium.		
1875	3	13	75
Total Sum	86	410	1641

The Radiation-points are deduced as follows: the right ascension and declination for the beginning and the end of each shooting star were laid down in a chart, constructed for this purpose. The charts are on the gnomonic projection, and large enough for estimated 0°·5 at 45° declination, and 1° at 85° declination. The path of each shooting star was drawn with a sharp pencil with as much precision as possible, and after this operation the paths were prolonged backwards, until several paths met in or near a point, and the 410 Radiation-points were thus determined upon the chart. The shooting stars used for the determination were within at most four days of each other; but it was only in the case of very poor showers that so long an interval was taken.

The Radiation-points were laid down on a new chart, and from those near each other the 80 probable Radiants were calculated by the method of least squares.

In the following three tables, Table I. contains the Radiation-points deduced from the O'Gyalla observations; Table II., those from the other stations. In each of these tables the first column shows from how many meteors the Radiation-point was deduced: the second and third give the R.A. and Declination of the Radiation-point. The left-hand three columns are to be read separately from, and as preceding, the right-hand three columns.

The two tables show Radiation-points for—

1. *February*, 20 and 24, observed at O'Gyalla.
2. *April*: April 19, 20 and 21, 1874; April 19, 20 and 21, 1878, from O'Gyalla; 1876, April 20, 22, from Agram; and 1878, April 20, 21, from Schemnitz.
3. *July*: 1873, July 25, 26, 27, 28 and 29; 1875,

July 26, 27, 28, 29 and 31; 1876, July 26, 27 and 28; 1877, July 27; and 1878, July 28 and 29, from O'Gyalla. For the same shower: 1875, July 25, 26, 27 and 28, from Schemnitz; 1876, July 26 and 27; 1878, July 25 and 26 from Schemnitz; 1875, July 25 and 27, from Agram; and 1878, July 28, from Hodmezövásárhely.

4. *August*: 1871, August 9; 1872, August, 6, 7, 9, 10, 11, 12 and 13, from O'Gyalla; 1873, August 8, 9, 10; 1874, August 8, 10, 11 and 12; 1875, August 8, 9, 10, 11 and 12; 1876, August 12, 13, and 14; 1877, August 12 and 13; and 1878, August 9, 10 and 12, also from O'Gyalla; 1875, August 10, 11 and 12, from Szathmár; 1876, August 12 and 13; 1877, Aug. 10, from Hodmezövásárhely; 1877, August 11, 12 and 13, from Agram.
5. *October*: 1872, October 20, 22, 25 and 27; 1873, October 22, from O'Gyalla.
6. *November*: 1872, November 28; and finally, 1873, November 12, from O'Gyalla.

Table III. contains the deduction of the 80 probable Radiants: the first column gives the current Nos., 1 to 80; the second column shows the Radiation-points used in the determination: viz. these are referred to by means of the current Nos. of the observations for the year and day or days, as appearing by the first and second tables; the third and fourth columns give the R.A. and Declination of the probable Radiants.

TABLE I. (O'Gyalla.)

No. of Meteors.	Radiation-points. R.A.	Decl.	No. of Meteors.	Radiation-points. R.A.	Decl.
		1871. *February* 20, 24. Nos. 1 to 3.			
3	50·0	56·0	3	205·0	31·0
3	90 0	50·0			
		1874. *April* 19, 20, 21. Nos. 1 to 5.			
3	223·5	59·5	4	267·0	37·0
3	229·0	67·5	3	269·5	28·0
3	259·0	19·0			
		1878. *April* 19, 20, 21. Nos. 1 to 4.			
3	216·0	60·0	4	257·0	43·0
3	217·5	55·5	4	264·0	23·5
		1873. *July* 25, 26. Nos. 1 to 9.			
4	3·0	67·5	6	312·0	−8·0
7	212·5	57·5	6	323·5	3·5
6	279·0	81·5	6	327·0	25·0
6	302·0	41·0	4	351·0	64·0
18	304·0	60·0			

1873. *July* 27. Nos. 10 to 14.

No. of Meteors.	Radiation-points. R.A.	Decl.	No. of Meteors.	Radiation-points. R.A.	Decl.
	°	°		°	°
6	274·0	74·0	6	331·5	5·5
7	299·0	69·5	4	351·0	61·5
4	320·0	0·0			

1873. *July* 28, 28. Nos. 15 to 18.

5	0·0	90·0	4	325·0	65·0
5	316·0	−10·0	5	328·0	22·0

1875. *July* 26, 27, 28, 29. Nos. 1 to 14.

4	1·0	35·5	6	310·0	43·0
5	2·5	63·5	3	310·0	71·0
4	282·0	54·5	4	313·0	45·0
5	288·0	50·0	4	315·0	45·0
4	297·5	3·5	4	315·0	72·0
3	308·0	40·0	5	321·0	36·5
6	308·0	51·0	4	335·0	3·0

1875. *July* 31. Nos. 15 to 24.

5	276·0	39·0	6	316·5	17·0
3	293·0	−3·0	3	318·0	19·0
4	298·0	−8·0	5	332·0	24·5
3	213·0	30·0	4	339·0	27·0
3	315·0	31·5	4	353·0	47·5

1876. *July* 26, 27, 28. Nos. 1 to 15.

3	4·0	65·5	5	325·0	74·5
3	6·5	63·5	5	328·0	44·5
3	9·5	67·0	4	338·5	41·0
4	195·0	75·0	4	339·0	41·0
4	287·5	44·0	4	342·5	75·5
5	288·0	46·0	5	352·0	69·5
4	308·0	57·0	3	359·0	67·0
5	312·0	41·5			

1877. *July* 27. Nos. 1 and 2.

4	51·5	59·0	4	68·0	85·0

1878. *July* 28, 29. Nos. 1 to 11.

No. of Meteors.	Radiation-points. R.A.	Decl.	No. of Meteors.	Radiation-points. R.A.	Decl.
	°	°		°	°
3	100·0	84·0	6	288·0	45·5
3	132·0	83·0	3	304·0	−4·5
3	284·0	30·0	3	306·0	−2·5
3	287·0	30·0	4	349·0	18·0
5	287·5	3·0	5	352·0	13·0
3	288·0	33·0			

1871. *August* 9. Nos. 1 to 10.

4	22·0	71·5	3	242·0	48·0
4	96·0	71·0	4	247·0	50·5
3	140·0	83·0	3	266·0	44·5
3	161·0	83·5	4	268·0	60·0
3	179·9	79·5	4	271·0	44·0

1872. *August* 6, 7. Nos. 1 to 5.

3	300·0	39·0	4	337·0	43·0
3	306·0	45·5	3	342·0	52·0
3	309·5	42·5			

1872. *August* 9. Nos. 6 to 19.

4	6·0	4·0	5	324·5	59·0
5	249·0	61·0	4	330·0	76·0
4	265·5	28·5	3	343·0	60·0
4	270·0	57·0	3	346·5	48·0
7	276·5	14·5	4	346·5	59·0
4	298·5	56·0	3	347·5	47·0
4	312·0	46·0	3	351·0	58·5

1872. *August* 10. Nos. 20 to 38.

4	8·0	67·5	7	326·0	73·0
5	41·0	58·0	5	340·0	68·0
3	43·5	58·0	4	341·0	38·0
4	248·5	25·0	4	342·5	56·0
4	294·0	28·0	4	343·0	52·5
4	300·0	26·0	4	343·0	66·5
5	309·0	56·0	6	343·0	66·5
4	311·0	48·5	4	353·5	57·5
6	322·0	69·5	7	355·0	68·0
4	323·5	38·0			

1872. *August* 11. Nos. 39 to 46.

No. of Meteors.	Radiation-points. R.A.	Decl.	No. of Meteors.	Radiation-points. R.A.	Decl.
	°	°		°	°
7	295·0	60·0	5	334·5	44·5
5	297·0	20·5	5	336·0	42·0
7	313·5	8·5	5	338·0	45·0
4	330·0	60·0	3	340·0	45·0

1872. *August* 12, 13. Nos. 47 to 59.

3	4·0	45·0	3	331·0	63·5
3	8·5	47·5	3	340·5	44·0
3	11·5	46·5	3	352·0	52·0
4	28·5	48·5	3	355·0	46·0
4	32·0	63·0	3	358·0	40·0
3	304·0	50·0	4	95·0	87·0
3	328·0	59·0			

1873. *August* 8, 9, 10. Nos. 1 and 2.

3	19·0	65·0	3	290·0	49·0

1874. *August* 8, 10. Nos. 1 to 40, and (next page) Nos. 41 to 56.

3	0·0	55·0	6	38·5	58·5
3	1·0	53·0	5	39·0	48·5
3	1·0	52·0	3	39·0	66·0
3	3·0	53·0	4	40·0	68·5
4	9·5	40·5	3	42·0	66·0
5	11·0	46·0	4	43·0	68·5
5	18·5	47·5	4	46·5	58·5
3	24·0	66·5	4	48·0	63·0
3	25·0	65·5	6	50·0	84·0
3	27·0	67·0	5	54·0	69·5
3	28·0	65·0	4	63·0	67·0
3	29·5	67·5	4	63·0	68·5
4	31·0	49·5	4	64·5	59·7
5	32·0	48·0	5	72·0	64·0
4	32·0	59·7	6	78·0	66·5
3	33·0	67·0	4	86·0	65·0
4	36·0	77·5	4	170·0	86·0
6	37·0	58·0	4	273·0	47·5
4	38·0	63·0	3	321·5	49·5
4	38·5	67·5	4	323·0	46·0

No. of Meteors.	Radiation-points. R.A.	Decl.	No. of Meteors.	Radiation-points. R.A.	Decl.
	°	°		°	°
3	325·0	49·5	3	355·0	52·0
7	325·5	18·0	3	357·0	51·5
4	326·0	16·0	4	357·0	39·5
4	326·0	−13·0	4	358·0	52·0
5	343·0	50·5	3	358·0	50·5
5	345·0	41·0	3	358·0	53·5
7	346·0	46·5	5	359·5	74·0
4	350·2	27·0	5	359·5	42·0
	1874. *August* 11, 12. Nos. 57 to 63.				
4	217·0	73·0	3	290·0	52·0
3	250·0	73·0	4	292·0	48·0
3	263·0	67·0	5	337·0	70·0
3	270·0	72·0			
	1875. *August* 8. Nos. 1 to 4.				
3	18·0	73·5	3	29·0	71·5
3	21·0	77·5	3	323·0	72·0
	1875. *August* 9. Nos. 5 to 21.				
5	8·0	48·5	3	309·0	31·5
6	28·0	51·0	3	326·0	56·0
4	58·0	63·5	5	328·0	55·0
4	62·0	68·5	3	330·0	56·0
4	260·0	66·0	3	330·0	54·0
5	279·0	17·5	3	332·0	52·5
4	298·5	55·5	3	335·0	56·0
3	307·0	33·0	5	355·0	52·5
4	308·0	43·5			
1875. *August* 10. Nos. 22 to 37, and (next page) Nos. 38 to 55.					
4	6·0	77·5	5	40·5	63·0
4	22·5	37·0	6	46·5	52·0
5	22·5	52·5	5	47·0	50·5
7	28·0	49·5	3	48·5	51·0
5	28·5	55·0	5	52·0	67·0
5	32·0	57·5	4	63·5	66·0
5	34·5	55·0	6	65·0	83·0
6	38·0	67·0	5	85·0	72·5

No. of Meteors.	Radiation-points. R.A.	Decl.	No. of Meteors.	Radiation-points. R.A.	Decl.
	°	°		°	°
4	88·5	67·0	4	317·0	73·0
4	248·0	55·5	6	322·0	47·0
4	250·0	52·5	4	322·0	82·0
5	268·0	43·5	7	326·5	58·0
4	270·0	49·5	4	327·0	48·5
5	274·0	47·5	6	337·0	77·0
5	280·0	83·5	5	339·0	70·5
4	282·0	45·0	6	343·0	76·5
6	296·0	43·0	4	348·0	78·0
	1875. *August* 11, 12. Nos. 56 to 60.				
3	39·5	43·0	3	257·0	47·0
5	40·0	79·5	3	258·5	50·0
4	57·0	79·5			
	1876. *August* 12, 13, 14. Nos. 1 to 7.				
5	102·0	61·7	5	271·5	56·5
5	216·5	55·5	4	303·0	48·0
5	220·0	56·0	4	310·0	75·0
5	254·0	36·5			
	1877. *August* 12, 13. Nos. 1 to 5.				
4	0·0	84·5	4	9·0	45·0
4	3·0	50·0	4	36·5	61·5
4	7·0	55·5			
	1878. *August* 9, 10, 12. Nos. 1 to 3.				
3	2·5	41·0	4	358·0	39·0
4	38·0	67·0			
	1872. *October* 20, 22, 25, 27. Nos. 1 to 9.				
3	5·0	38·5	3	325·0	67·5
3	243·0	42·0	3	330·0	53·0
3	276·0	50·0	3	337·5	64·0
3	302·5	35·0	3	341·0	20·0
4	325·0	62·5			

1873. *October* 22. No. 1.

No. of Meteors.	Radiation-points. R.A.	Decl.	No. of Meteors.	Radiation-points. R.A.	Decl.
	°	°			
3	47·0	53·5			

1872. *November* 21. Nos. 1 to 3.

3	77·5	50·0	4	333·5	59·0
3	319·0	68·0			

1873. *November* 12. Nos. 1 and 2.

4	335·5	28·5	3	351·0	45·0

Table II. (Other Stations.)

AGRAM.

1876. *April* 20, 22. Nos. 1 to 4.

3	181·0	55·0	3	225·0	46·0
3	224·0	−3·0	3	272·0	43·5

1875. *July* 25, 27. Nos. 1 to 11.

4	3·5	61·0	3	314·0	− 8·5
4	16·0	81·0	3	320·0	66·5
3	300·0	59·0	3	322·0	− 6·0
3	305·0	−11·5	3	326·5	8·0
3	309·5	− 7·5	5	350·5	56·5
3	313·5	4·5			

1877. *August* 11, 12, 13. Nos. 1 to 10.

6	249·0	9·0	5	276·0	− 7·5
4	251·8	−24·0	4	278·0	−15·2
5	260·5	5·0	6	288·0	4·0
4	270·0	1·5	5	296·0	19·0
5	272·5	−20·2	8	309·0	13·5

SCHEMNITZ.

1875. *July* 25, 26, 27, 28. Nos. 1 to 8.

4	276·0	15·5	4	343·0	57·5
4	282·5	−15·0	4	344·0	20·0
6	308·0	− 3·5	4	345·5	56·5
4	334·0	57·5	6	352·5	19·0

1876. *July* 26, 27. Nos. 1 to 6.

No. of Meteors.	Radiation-points. R.A.	Decl.	No. of Meteors.	Radiation-points. R.A.	Decl.
	°	°		°	°
4	105·0	71·0	4	245·0	11·5
3	232·0	17·0	3	273·5	41·0
3	237·5	30·0	3	274·5	16·0

1878. *July* 25, 26. Nos. 1 to 3.

4	263·0	52·5	3	279·0	46·0
3	271·0	36·0			

SZATHMÁR-NÉMETHI.

1875. *August* 10, 11, 12. Nos. 1 to 13.

4	5·0	21·0	7	46·5	50·0
5	8·5	16·5	12	49·5	58·5
5	17·0	28·5	8	63·0	46·0
8	18·0	8·0	4	59·0	25·0
4	23·0	58·5	4	71·0	53·0
8	38·0	53·5	4	338·0	71·5
4	44·0	41·5			

HODMEZÖ-VÁSÁRHELY.

1878. *April* 20, 21. Nos. 1 to 5.

3	249·0	23·0	7	275·0	40·5
3	249·0	27·5	4	303·0	47·5
4	256·0	23·5			

1878. *July* 25. Nos. 1 to 3.

3	90·0	90·0	4	329·0	45·5
3	327·0	26·0			

1876. *August* 9, 10, 11. Nos. 1 to 20.

5	4·5	44·0	5	60·0	78·5
4	5·0	2·5	4	72·0	72·0
4	9·0	2·0	4	90·0	90·0
4	7·0	40·0	3	139·0	59·0
4	12·5	25·0	5	320·0	70·0
4	20·0	67·0	4	330·0	58·0
5	22·5	59·0	4	340·0	30·0
6	43·0	63·0	5	346·0	41·0
5	51·5	56·5	4	347·0	46·0
6	60·0	77·5	3	348·0	43·0

1876. *August* 12, 13. Nos. 1 to 9.

No. of Meteors.	Radiation-points. R.A.	Decl.	No. of Meteors.	Radiation-points. R.A.	Decl.
4	31°·0	58°·5	3	330°·0	88°·0
4	43·0	59·0	3	337·0	49·0
3	269·0	21·5	4	347·0	61·0
4	283·5	18·0	4	358·0	51·5
3	326·0	46·0			

1877. *August* 10. Nos. 1 and 2.

3	173·0	76·0	3	212·5	70·5

Table III. (The 80 probable Radiants.)

1874. *April* 19, 20, 21.

No. of Radiant.	Nos. of the Radiation-points.	Position of Radiant. R.A.	Decl.
1	1, 2	226°·3	63°·5
2	4, 5	268·1	33·1
	1878. *April* 19, 20, 21.		
3	1, 2	216·8	57·3
	1873. *July* 25, 26.		
4	1, 9	357·0	65·7
	1873. *July* 27.		
5	10, 11	287·5	71·6
	12, 13	326·9	3·3
	1875. *July* 26, 27, 28, 29.		
7	3, 4	285·3	52·0
8	6, 7, 8, 10, 11	310·3	45·4
9	9, 12	312·8	71·6
	1875. *July* 31.		
10	16, 17	295·9	− 5·8
11	18, 19	314·0	30·7
12	20, 21	317·0	17·7
13	22, 23	335·1	25·6
	1876. *July* 26, 27, 28.		
14	1, 2, 3, 14, 15	1·0	66·8
15	5, 6	287·8	45·1
16	10, 11, 12	334·6	42·3
17	9, 13, 14	339·6	73·0

1871. *July* 28, 29.

No. of Radiant.	Nos. of the Radiation-points.	Position of Radiant. R.A.	Decl.
		°	°
18	3, 4, 6	286·3	31·0
19	8, 9	305·0	− 3·5
20	10, 11	350·5	15·5

1871. *August* 9.

21	3, 4, 5	160·0	82·0
22	8, 10	268·8	44·2

1872. *August* 6, 7.

23	1, 2, 3	305·2	42·3
24	4, 5	339·1	46·9

1872. *August* 9.

25	15, 16, 17, 18, 19	346·9	54·8

1872. *August* 10.

26	21, 22	41·9	58·0
27	24, 25	297·0	27·0
28	26, 27	309·9	52·7
29	28, 30, 31, 35, 36	333·8	69·0
30	33, 34, 37	346·3	55·3

1872. *August* 11.

31	39, 42	307·7	60·0
32	42, 43, 44, 45, 46	336·9	44·0

1872. *August* 12, 13.

33	47, 48, 49	8·0	46·3
34	53, 54	329·5	61·3
35	56, 57, 58	355·0	46·0

1874. *August* 8, 10.

36	5, 6, 7	13·3	44·9
37	13, 14	31·6	48·7
38	7, 8, 9, 10, 11, 16, 19, 20, 23, 24, 25, 26, 27, 28	36·6	65·6
39	15, 18, 21, 22, 27	38·5	56·6
40	31, 32	63·0	67·8
41	34, 35	75·3	65·4
42	39, 40, 41	323·2	48·1
43	1, 2, 3, 4, 49, 50, 52, 53, 54,	359·0	52·5

1874. *August* 11, 12.

No. of Radiant.	Nos. of the Radiation-points.	Position of Radiant. R.A.	Decl.
		°	°
44	58, 59, 60	261·0	70·7
45	61, 62	291·1	49·7
	1875. *August* 8.		
46	1, 2, 3	22·7	74·2
	1875. *August* 9.		
47	12, 14	308·0	32·2
48	15, 16, 17, 18, 19, 20	330·0	54·9
	1875. *August* 10.		
49	26, 27, 28	31·7	55·8
50	29, 30	39·1	65·2
51	31, 32, 33	47·1	51·3
52	34, 35	57·1	66·5
53	39, 40	249·0	54·0
54	41, 42, 43	270·7	46·6
55	48, 51	324·0	47·6
56	53, 54, 55	341·3	75·5
	1875. *August* 11, 12.		
57	59, 60	257·8	48·5
	1876. *August* 12, 13, 14.		
58	2, 3	218·3	55·8
	1877. *August* 12, 13.		
59	2, 3, 4	6·3	50·2
	1872. *October* 20, 22, 25, 27.		
60	5, 6, 7, 8	329·0	61·8
	1872. *November* 28.		
61	1, 2, 3	327·3	62·9
	ZÁGRÁB (AGRAM).		
	1875. *July* 25, 27.		
62	4, 5, 6, 7, 9, 10	315·1	− 3·5

1877. *August* 11, 12. 13.

No. of Radiant.	Nos. of the Radiation-points.	Position of Radiant. R.A.	Decl.
		°	°
63	3, 4, 6, 7	270·8	− 3·7
64	8, 9, 10	298·5	11·9

SCHEMNITZ.

1875. *July* 25, 26, 27, 28.

65	4, 5, 7	340·8	57·2
66	6, 8	349·1	19·4

1878. *July* 25, 26.

67	1, 2, 3	270·2	45·6

SZATHMÁR-NÉMETHI.

1875. *August* 10, 11, 12.

68	1, 2, 3, 4	15·1	16·1
69	5, 6	40 0	49·5
70	7, 8, 9	52·7	52·6

HÓDMEZÖ-VÁSÁRHELY.

1876. *August* 9, 10, 11.

71	1, 4	5·6	42·2
72	2, 3	7 0	2·3
73	6, 7	21·4	62·6
74	8, 9	46 9	60·0
75	10, 11, 12	63·2	76·4
76	17, 18, 19	345·1	39·9

1876. *August* 12, 13.

77	1, 2	7·0	58·8
78	5, 7	31·5	47·5
79	8, 9	352·5	56·3

1878. *April* 20, 21.

80	1, 2, 3	251·8	24·0

Meteors observed during Voyage from London to Melbourne and home.

By Mr. D. W. Barker, 2nd. Officer on board ship 'Superb.'

Explanation of Symbols used in "Mag." Column.

... denotes very brilliant.
× ,, brightness of 1st mag. stars.
\+ ,, 2nd mag. stars. ^ added shows it left a stream behind.
— ,, all below 2nd mag.

Date		Time.	Lat.	Long.	From ✶	To ✶	No.	Mag.
1879, July	28	...	7 S	25 W	δ Cephei	β Cephei	1	—
	28	...	,,	,,	Between β & γ Trianguli	16 Persei	1	—
	28	...	,,	,,	ι Ceti	Mira	2	—
	28	...	,,	,,	κ Fornacis	δ Horologii	2	—
	28	...	,,	,,	κ Fornacis	η Horologii	1	—
	28	...	,,	,,	α Doradûs	γ Pictoris	1	—
	28	...	,,	,,	52 Arietis	through Pleiades	1	—
	28	...	,,	,,	α Reticuli	α Pictoris	1	—
	28	...	,,	,,	between ε & τ Eridani	between ν & 54 Eridani	1	—
	28	...	,,	,,	ν Ceti	τ Eridani	1	—
	28	...	,,	,,	1 Eridani	ζ Phœnicis	1	×
	28	...	,,	,,	ι Eridani	between ν^1 & ν^3 Eridani	1	×
	28	...	,,	,,	α Eridani	α Hydri		1<
	28	...	,,	,,	41 Arietis	South of Pleiades	1	

Meteors numerous, average from 0 to 4 A.M. 3 in a minute.

Date.		Time.	Lat.	Long.	From ✱	To ✱	No.	Mag.
1879, July	29	...	10 S	27 W	ϵ Sculptoris	α Eridani	1	—
	29	...	,,	,,	δ Phœnicis	ϕ Eridani	1	—
	29	...	,,	,,	γ Phœnicis	κ Eridani	1	—
			Meteors numerous, average from 0 to 4 A.M. 2 in a minute.					
Sept.	13	...	41 S	118 E	between α & μ Canis maj.	5 Monocerotis	1	+
	15	...			Very few meteors.			
1880, Feb.	5	...	35 S	30 W	λ Leonis	North of γ Cancri	1	+
	5	...			Very few meteors.			
	10	1.30 A.M.	26 S	34 W	ρ Coronæ	12 Boötis	1	+<
	10	2.30 A.M.	,,	,,	μ Centauri	γ Muscæ	1	|<
	10	3.35 A.M.	,,	,	Spica	ϕ Ophiuchi	1	×<
	10	...	,,	,,	12 Boötis	θ Coronæ	1	—
	11	3.20 A.M.	25 S	33 W	69 Boötis	9 Boötis	1	+<
	12	3.0 A.M.	,,	,,	23 Comæ	between 2 & 6 Canum Venaticorum	1	|<
	12	4.0 A.M.	,,	,,	Scorpionis	θ Ophiuchi	1	|<
	14	1.30 A.M.	23 S	,,	α Canum Venat.	83 Ursæ maj.	1	—
	14	2.15 A.M.	,,	,,	X Argûs	π^1 Columbæ	1	—
	16	2.0 A.M.	19 S	32 W	δ Crucis	η Chamæleontis	1	+<
	16	7.0 P.M.	,,	,,	ζ Persei	ν Persei	1	×<
	16	7.30 P.M.	,,	,	ϵ Volantis	κ Argûs (Carina)	1	|<

Date.	Time.	Lat.	Long.	From ✳	To ✳	No.	Mag.
1880, Feb. 18	0.30 A.M.	16 S	32 W	ϵ Centauri	225 (neb) Lupi	1	—
19	1.40 A.M.	14 S	31 W	ρ Serpentis	β Herculis	1	—
19	3.15 A.M.	,,	,,	N. of β Scorpionis	N. of Antares	1	—
19	3.30 A.M.	,,	,,	χ Virginis	16 Leonis min.	1	+
20	2.0 A.M.	11 S	30 W	μ Centauri	3496 Centauri	1	—
21	...	10 S	30 W	51 Herculis	68 Herculis	1	—
March 5	2.0 A.M.	19 N	34 W	ζ Herculis	θ Draconis	1	—
				Very few meteors.			
7	1.0 A.M.	23 N	37 W	ι Centauri	η Centauri	1	—
9	0.45 A.M.	23 N	40 W	δ Urs. maj.	towards 1625 Σ Urs. min.	1	—
9	...		Through	5 Urs. min.	5° each way	1	+
9	1.0 A.M.	,,	40 W	ν^1 Draconis	30 Draconis	1	—
9	1.30 A.M.	,,	,,	ζ Herculis	68 Herculis	1	—
9	3.30 A.M	,,	,,	37 Urs. maj.	38 Urs. maj.	1	—
				Numerous meteors.			
10	1.0 A.M.	24 N	39 W	δ Urs. min.	76 Draconis	1	—
				Very few meteors.			
11	1.0 A.M.	,,	,,	between β & γ Urs. min.	Towards Polaris	1	—
11	2.30 A.M.	,,	,,	δ Scorpionis	3 Centauri	1	—
11	2.40 A.M.	,,	,,	δ Draconis	α Cephei	1	—

Date.	Time.	Lat.	Long.	From ✱	To ✱	No.	Mag.
1880, March 11	2.50 A.M.	24 N	39 W	ξ Draconis	Σ 2403 Draconis	1	—
11	3.0 A.M.	,,	,,	ϵ Ophiuchi	η Scorpionis	1	×<
11	3.0 A.M.	,,	,,	45 Draconis	15 Cygni	1	—
11	6.40 P.M.	,,	,,	α Piscium	? double 33 H Piscium	1	×<
12	2.45 A.M.	24 N	39 W	γ Urs. min.	4 Urs. min.	1	—
12	3.0 A.M.	,,	,,	ξ Urs. maj.	μ Urs. maj.	1	><
12	7.15 A.M.	,,	,,	γ Arietis	64 Piscium	1	×<
13	3.15 A.M.	$24\frac{1}{2}$ N	,,	δ Urs. min.	Polaris	1	+
				Very few meteors.			
14	1.0 A.M.	25 N	,,	δ Urs. min.	28 Cephei	1	—
18	1.15 A.M.	28 N	37 W	φ Urs. maj.	52 Cameli	1	—
18	3.15 P.M.	,,	,,	60 Leonis	between ρ & α Leonis	1	—
				Few meteors.			
21	2.0 A.M.	31 N	31 W	σ Scorpionis	χ Lupi	1	×
21	3.0 A.M.	,,	,,	φ Urs. maj.	θ Urs. maj.	1	+

Meteorological Office,
1880, *April* 5.

On the Theoretical Value of the Acceleration of the Moon's Mean Motion in Longitude, produced by the Change of Excentricity of the Earth's Orbit. By Sir G. B. Airy, K.C.B. Astronomer Royal.

1. In the *Monthly Notice* of the Royal Astronomical Society for 1874, January, I explained the principles of a novel method of treating the Lunar Theory. My object was to dispense with long algebraic expansions, far as as possible. For this purpose, I proposed to use the numerical coefficients found by a preceding investigator (Delaunay), not as accurate, but as very approximate, requiring numerical corrections (treated, in the first instance, in a symbolical form) so small that multiples of those symbols (when reduced to numbers) to the first power only would suffice for the correction of the numerical value of every function depending on the coefficients. The immediate object was to find the numerical relation between the theoretical movements of the Moon and the forces (themselves depending on those movements) which ought to account for the movements; and to find whether, by varying the movements and varying in correspondence therewith the forces which depend on those movements, the relation between the movements and the forces can be made perfect.

2. It is evident that this form of investigation is not limited to the correction of assumed coefficients, but can also be applied to the examination of the effects of introducing small forces not contemplated in the original theory. These small forces may in some instances depend on geometrical considerations (as in the effect of the Earth's oblateness), in other cases they may depend on time (as in the slow change of solar elements producing certain disturbances in the motions of the Moon). In such cases, the changes from the original suppositions are in fact the introduction of new forces; and here we have to consider how, by varying the movements and varying in correspondence therewith the forces which depend on those movements (as stated in the first paragraph), and also introducing the new forces and their variations depending on the movements, the relation between the movements and the forces can be made perfect. The original fundamental supposition must, however, be maintained, that the new variations of forces and movements are to be so small that the first power of the algebraic terms expressing them will suffice.

3. The units employed in the further investigation which I am now to explain (the same which are used in my general Lunar Theory) are the following:—

The unit of longitudinal measure is the mean distance of the Moon from the Earth at the epoch (say A.D. 1900). In respect of absolute measure, that distance is called 1, but in respect of variability, it is called a. In the present investigation, we shall not need to consider the variability.

The unit of angular measure is the angle corresponding to an arc of a circle whose length is equal to radius 1. It is equal, in degrees, to $\frac{360^\circ}{2\pi}$ or, in seconds, to $\frac{1296000''}{2\pi}$.

The unit of time is the time occupied by the Moon, with her mean angular motion at the epoch, in describing the unit of angle. We shall have to compare it with the length of a Julian year: it is easily seen that, as referred to our unit, the Julian year =

$$\frac{\text{Moon's mean angular motion}}{\text{Sun's mean angular motion}} \times 2\pi = 13{\cdot}368753 \times 2\pi = 83{\cdot}998352.$$

The unit of the Moon's angular motion is therefore necessarily the angular velocity with which the Moon in mean angular motion describes the angle 1 in the time 1. In respect of absolute measure of angular motion, this is called 1; in respect of variability (not, however, considered here), it is called n.

4. The Moon's radius vector, the projection of radius vector on the plane of the ecliptic, her longitude, her latitude, and all combinations of these, are (in the most general theory) expressed in indefinite series of periodical terms, whose arguments consist of sums of various multiples (sometimes positive, sometimes negative) of the following quantities:—

D = excess of Moon's mean longitude above Sun's mean longitude.
f = excess of Moon's mean longitude above mean longitude of node.
l = excess of Moon's mean longitude above mean longitude of perigee.
S = mean anomaly of Sun.

These are all treated as increasing uniformly with the time.

In the present investigation, however, it is proposed to neglect the variable terms related to the excentricity of the Moon's orbit and its inclination to the ecliptic, not as non-existent, but as producing nothing of sensible amount additive to the terms employed in the present inquiry; and thus the symbols f and l will not appear. The modifications which they produce in the constant term will, in the first instance, be retained. We shall not treat of the disturbance of the Moon's latitude. As regards the excentricity of the Sun's orbit, we shall use a mean of the Sun's effect through each year; and thus S will not be required. Thus the only argument remaining is D, and the only multiple of that argument which we shall use is 2D, of which the effects far exceed those of D or 3D. And for 2D we shall write F.

5. The equations which we use in the general theory to determine the Moon's motion apply to T (a force in the plane of the Sun's orbit, normal to the projection of the Moon's radius

vector, and accelerating the Moon), P (a force in the same plane, in the projection of the radius vector, and directed from the Earth), and Z (a force normal to the plane of the Sun's orbit, in the direction in which z is considered positive). Z, however, has here no existence, as the inclination of the two orbits is neglected.

6. By investigations of which the introductory parts are printed, a "Factorial Table" has been prepared, exhibiting in a symbolical form, as factors of the lunar geometrical disturbances, the forces which must be introduced as acting on the Moon in order to explain the disturbed movements of the Moon. Any possible disturbances, subject to the limitation above mentioned in regard to inclination of orbit, may be expressed by disturbances of $\frac{a}{r}$ and v, where r is the radius vector of the Moon, and v the Moon's true longitude. For such disturbances we put the symbols $\delta\frac{a}{r}$ and δv. Then the Factorial Table, limited as is above mentioned, gives the following equations, in which the value of $\frac{r}{a}$ is $1-\cdot0077\cos F$:

$$T\frac{r}{a}=\left\{\begin{array}{llll}+\delta\left(\frac{a}{r}\right) & \times(& \cdot0000 & -\cdot0281\sin F)\\ +\frac{d}{dt}\left(\delta\frac{a}{r}\right) & \times(-1\cdot9919 & & +\cdot0061\cos F)\\ +\delta v & \times(& \cdot0000 & +\cdot0168\cos F)\\ +\frac{d}{dt}(\delta v) & \times(& \cdot0000 & +\cdot0268\sin F)\\ +\frac{d^2}{dt^2}(\delta v) & \times(+1\cdot0006 & & -\cdot0144\cos F)\end{array}\right\}$$

$$P\frac{r}{a}=\left\{\begin{array}{llll}+\delta\left(\frac{a}{r}\right) & \times(+2\cdot9964 & & -\cdot0236\cos F)\\ +\frac{d}{dt}\left(\delta\frac{a}{r}\right) & \times(& \cdot0000 & -\cdot0528\sin F)\\ +\frac{d^2}{dt^2}\left(\delta\frac{a}{r}\right) & \times(-1\cdot0053 & & +\cdot0212\cos F)\\ +dv & \times(& \cdot0000 & +\cdot0168\sin F)\\ +\frac{d}{dt}(\delta v) & \times(-1\cdot9888 & & -\cdot0090\cos F)\end{array}\right\}$$

These equations give, in the first instance, the values of forces which can produce an assigned geometrical disturbance; our object, however, will be, by a reversal-process, to find the geometrical disturbances which will be required to correspond to an assigned external force.

7. It is important to observe that, in forming these equations, the variations of terrestrial attraction and of solar attraction,

depending on the change of the Moon's place produced by the disturbance of which we treat, are accurately taken into account, whatever the change of place may be.

We now proceed to compute the Disturbing Forces.

8. Taking the mean parallaxes of the Sun and Moon as $8''{\cdot}91$ and $3422''{\cdot}3$, the following values are obtained by known developments:—

Mean tangential disturbing force at epoch $\quad -{\cdot}008284 \,.\, \sin F.$

Mean radial disturbing force at epoch $\quad +{\cdot}002683 + {\cdot}008245 \,.\, \cos F.$

(These numbers have been computed in a general process, embracing all the inequalities of the lunar motions.)

Now suppose that, in the progress of time, the solar energy which produces these forces varies from the force at tabular epoch in the proportion of $1 : 1 + bt$. This is the same thing as introducing new forces, whose values are equal to the values above-given, multiplied by bt. Thus we obtain—

$$T = -{\cdot}008284 \times bt \,.\, \sin F;$$
$$P = +{\cdot}002683 \times bt + {\cdot}008245 \times bt \,.\, \cos F;$$

and these quantities, multiplied by $1 - {\cdot}0077 \cos F$, are to be substituted on the left side of the equation in Article 6. But the multiplication of the small term by T, and by the second part of P, will produce terms depending on 2F, which are everywhere neglected. The multiplication of the first part of P, however, introduces $-{\cdot}002683 \times {\cdot}0027 \times bt \,.\, \cos F$. This is similar to the second part, and will be associated with it.

9. In future we shall adopt the following notation:—

For $\frac{dF}{dt}$ we shall write m: the numerical value of m is $+1{\cdot}8504$. For

${\cdot}008284$, ${\cdot}002683$, and ${\cdot}008245$ (numbers without signs),

we shall use A, B, C.

A and C are sensibly equal.

The equations of Article 6 now become—

Equation for the Moon's ecliptic longitude:—

$$-A.bt \,.\, \sin F = \left\{ \begin{array}{ll} +\delta\frac{a}{r} & \times(\ {\cdot}0000 - {\cdot}0281 \sin F) \\ +\frac{d}{dt}\left(\delta\frac{a}{r}\right) & \times(-1{\cdot}9919 + {\cdot}0061 \cos F) \\ +\delta v & \times(\ {\cdot}0000 + {\cdot}0168 \cos F) \\ +\frac{d}{dt}(\delta v) & \times(\ {\cdot}0000 + {\cdot}0268 \sin F) \\ +\frac{d^2}{dt^2}(\delta v) & \times(+1{\cdot}0006 - {\cdot}0144 \cos F) \end{array} \right\}$$

Equation for the projection of the Moon's radius vector :—

$$\left.\begin{matrix} +\mathrm{B}.bt. \\ +(\mathrm{C}-\cdot0077.\mathrm{B})bt.\cos\mathrm{F} \end{matrix}\right\} = \left\{\begin{matrix} +\delta\frac{a}{r} & \times(+2\cdot9964-\cdot0236\cos\mathrm{F}) \\ +\frac{d}{dt}\left(\delta\frac{a}{r}\right) & \times(\quad\cdot0000-\cdot0528\sin\mathrm{F}) \\ +\frac{d^2}{dt^2}\left(\delta\frac{a}{r}\right) & \times(-1\cdot0053+\cdot0212\cos\mathrm{F}) \\ +\delta v & \times(\quad\cdot0000+\cdot0168\sin\mathrm{F}) \\ +\frac{d}{dt}(\delta v) & \times(-1\cdot9888-\cdot0090\cos\mathrm{F}) \end{matrix}\right\}$$

In the following Articles we shall use C′ for (C − ·0077. B), and in future calculations we shall use the integral numbers −2, +1, +3, −1, −2, for the broken numbers in the first column, 2nd, 5th, 6th, 8th and 10th lines.

10. It is perhaps vain to expect that equations so complex as those exhibited above can be solved directly. The only hope to solve them rapidly and successfully must be in carefully selecting a form for the solution, and in substituting its terms with indeterminate coefficients. The form adopted has been established from the following considerations :—

(1.) Whatever takes place in the Moon's mean parallax will be repeated, lunation after lunation, in the same way. Thus $\delta\frac{a}{r}$ may have a term $+ct$.

(2.) A corresponding change will take place in the mean arc of longitude described in every portion of time, so that the alteration of mean arc described in one portion, which is the sum of all alterations in the portions up to that time, will be proportional to t; and the sum of these alterations of mean arc up to that time, which is the alteration of longitude, will be proportional to t^2. Thus δv may have a term ht^2.

(3.) and (4.) Since the Sun's motion is not altered, F, which = 2 Moon's longitude − 2 Sun's longitude, will receive the addition $2ht^2$; and this term will geometrically (not by multiplication of any other perturbation) modify the original quadrantal terms, so that $\delta\frac{a}{r}$ will have a term such as ft^2. sin F, and δv will have such a term as kt^2. cos F.

(5.) and (6.) During the progress of these changes, it is not unreasonable to expect that the quadrantal modification of the orbit may undergo changes proportional to the time; and it seems likely that $\delta\frac{a}{r}$ may require such a term as gt. cos F. and that δv may require lt. sin F.

(7.) and (8.) Something will depend on the time when we suppose the intrusion of the new force and the disturbance of movements to begin. It may happen that the new force will

not begin at Lunar Conjunction or Opposition. Our assumption $T = -Abt \sin F$ implies that there will never be a disturbing force in longitude when F has one special value + integral multiples of π. It appears that this can be well met by the term $e.\sin F$ in $\delta\frac{a}{r}$ and a constant term i in δv.

11. The following assumptions, therefore, have been made :—

$$\delta\frac{a}{r} = +c\,.\,t + e\,.\sin F + ft^2\,.\sin F + gt\,.\cos F;$$
$$\delta v = +ht^2 + i \qquad + kt^2\,.\cos F + lt\,.\sin F,$$

containing eight indeterminate quantities; and these expressions and their differentials have been substituted in the equations of Article 9. It is proper to remark that in the multiplication of series forming every step of this operation, sin F and cos F are most carefully preserved; but $\sin^2 F$, $\cos^2 F$, and sin F. cos F, are neglected; they all relate to the argument 2F or 4D, which has been omitted from the adopted portion of the Factorial Table; and their factors are all very small.

The results of substitution admit of being arranged in eight groups, as follows :—

(1) $-2c+2h = 0.$

(2) $(+3c-4h-\mathrm{B}b)t = 0.$

(3) $(-2mf+\cdot0168.h-m^2k)t^2\,.\cos F = 0.$

(4) $(+3f+m^2f+\cdot0168.h+2mk)t^2\,.\sin F = 0.$

(5) $(-\cdot0281.c-4f+2mg+\cdot0536.h-4mk-m^2l+\mathrm{A}b)t\,.\sin F = 0.$

(6) $(-\cdot0234.c-4mf+(m^2+3)g-\cdot180.h-4k-2ml-\mathrm{C}'b)t\,.\cos F = 0.$

(7) $(+\cdot0061.c-2me-2g-\cdot0288.h+\cdot0168.i+2k+2ml)\,.\cos F = 0.$

(8) $(-\cdot0528.c+(m^2+3)e-2f+2mg+\cdot0168.i-2l)\,.\sin F = 0.$

12. The solution of these equations, though troublesome, is not difficult. The following appear to be the results. In the last column, the expressions are simplified by considering A=3B, C= 3B :—

c =				$-\mathrm{B}b.$
e =				$+\cdot6195\,.\,\mathrm{B}b.$
f =				$+\cdot0144\,.\,\mathrm{B}b.$
g =	$+\cdot4485\ \mathrm{A}b$	$+\cdot0238\,.\,\mathrm{B}b$	$+\cdot4141\,.\,\mathrm{C}'b.$	$= +2\cdot6116\,.\,\mathrm{B}b.$
h =				$-\mathrm{B}b.$
i =				$-\ 370\cdot83\ .\,\mathrm{B}b.$
k =				$-\ \cdot0205\,.\,\mathrm{B}b.$
l =	$+\cdot7769\,.\,\mathrm{A}b$	$+\cdot0457\,.\,\mathrm{B}b$	$+\cdot4476\,.\,\mathrm{C}'b$	$= +3\cdot7192\,.\,\mathrm{B}b.$

It will be remembered that the signs of c, e, f, and g, are those which apply to $\delta\left(\frac{a}{r}\right)$. The signs are to be changed for $\delta\left(\frac{r}{a}\right)$.

The term which expresses the secular acceleration in longitude is $ht^2 = -Bb.t^2$. To the numerical value of this term we will now give attention.

13. First, we have to ascertain the value, in ordinary language, of B=·002683. The value is here expressed in decimal parts of the radius; we require it in seconds of arc. Now

$$2\pi = 360 \times 60 \times 60 \times 1'' = 1296000 \times 1'';$$

$$\text{therefore } \cdot 002683 = \frac{1296000''}{2\pi} \times \cdot 002683.$$

14. Secondly, to find the value of b in terms of ΔE. Let σ, A, E, R, V, T, be the Sun's mass, the orbital semiaxis major, the excentricity of the solar (or terrestrial) orbit, the orbital radius vector, the Sun's true longitude, and the Sun's periodic time (one year.) The mean effect (as regards special points of the Moon's orbit) of the Sun, in a given state of his orbit and a given position in his orbit, to disturb the Moon's motion, is represented by $\frac{\sigma a}{R^3} \times \Delta t$, Δt being an element of time. Let ΔV be the corresponding element of longitude. The area described by the radius vector is $\frac{1}{2}R^2\Delta V$. The whole area of the ellipse is $\pi A^2\sqrt{(1-E^2)}$. Then this proportion holds—

$$\Delta t : T :: \tfrac{1}{2}R^2\Delta V : \pi A^2\sqrt{(1-E^2)};$$

$$\text{and therefore } \Delta t = \frac{TR^2 . \Delta V}{2\pi A^2\sqrt{(1-E^2)}};$$

and the disturbing effect of the Sun in the element of time is

$$\frac{\sigma a}{R^3} . \frac{TR^2 . \Delta V}{2\pi A^2\sqrt{(1-E^2)}} = \frac{\sigma a T}{2\pi . A^2\sqrt{(1-E^2)}} . \frac{\Delta V}{R}.$$

Now

$$\Delta v = \Delta \text{ (true anomaly), and } R = \frac{A(1-E^2)}{1 + E . \cos \text{ (true anomaly)}};$$

therefore the disturbing effect

$$= \frac{\sigma a}{2\pi . A^3} \times \frac{1 + E . \cos \text{ (true anomaly)}}{1-E^2} \times \frac{T . \Delta \text{ (true anomaly)}}{\sqrt{(1-E^2)}}.$$

Integrating this through an entire orbital revolution of the Sun, the value is

$$\sigma a \times \frac{T}{A^3} \times \frac{1}{(1-E^2)^{\frac{3}{2}}} = \sigma a . \frac{T}{A^3}\left\{1 + \frac{3}{2}E^2 + \frac{3.5}{2.4}E^4 + \&c.\right\}.$$

The variation of this quantity depending on the variation δE is

$$+ \sigma a \frac{T}{A^3} \times \left\{3E + \frac{15}{2}E^3\right\}\delta E$$

and the proportion of the variation to the unvaried quantity is

$$+\frac{3E+\frac{15}{2}E^3}{1+\frac{3}{2}E^2} = +3E\times\frac{1+\frac{5}{2}E^2}{1+\frac{3}{2}E^2} = +3E(1+E^2)\delta E, \text{ nearly.}$$

15. Thirdly, to find the values of E and δE, and the proportion of the variation to the unvaried quantity just mentioned:

By Le Verrier's *Annales de l'Observatoire de Paris*, tome iv., page 103, E=·01676927 − ·0000004338 × number of years; and the proportion of the variation above mentioned, for one year, to the unvaried quantity, is

$$-3\times\cdot01676927\times\{1+(\cdot01677)^2\}\times\cdot0000004338.$$

To infer from this the proportion for one orbital revolution of the Moon, we must multiply by

$$\frac{\text{time of Moon's orbital revolution}}{\text{time of Sun's orbital revolution}}, \text{ or } \frac{\text{Sun's motion in longitude for 30 days}}{\text{Moon's motion in longitude for 30 days}}$$

$$\text{or } \frac{106445\cdot7}{1423046\cdot6}.$$

And to infer the proportion for our unit of time, we must further multiply by

$$\frac{\text{unit of time}}{\text{units of time in Moon's orbital revolution}} \text{ or } \frac{1}{2\pi}.$$

Thus we obtain for the proportion of the variation to the unvaried quantity during one unit of time, or for b,

$$-3\times\cdot01676927\times1\cdot0002812\times0000004338\times\frac{1064457}{2\pi\times14230466}.$$

16. And to obtain the value of δv for one year, we must multiply this by the square of $\frac{\text{time of Sun's orbital motion}}{\text{time of Moon's orbital motion}}\times$ units of time in Moon's orbital motion, or $\left(\frac{2\pi\times14230466}{1064457}\right)^2$. Thus we obtain finally for the value of δv or $-Bb.t^2$,

For one year, $0''\cdot00101477$.

For a century, $10''\cdot1477$.

I present this result to the Society with much confidence.

It is to be remarked that this numerical value is founded upon the numerical value attached to the solar parallax in the first step of the investigations—namely $8''\cdot91$. And the magnitudes of the quantities A,B,C, depend upon the inverse cube of the Sun's distance, or upon the cube of parallax. If, for instance,

the parallax be diminished by $\frac{1}{60}$ part, the magnitudes of A,B,C, and of the acceleration, will be diminished by $\frac{1}{20}$ part.

17. The following deductions are unimportant, but they may be interesting. In the Moon's orbit, $1'' = 6000$ feet, very nearly. In a century, therefore, the Moon is accelerated 60000 feet. In the first year, the acceleration is 6 feet; in the second year, 18 feet additional, &c. This is additive to the computed longitude, whether before or after the epoch.

In a century, the Moon's distance is changed by $\frac{60000 \text{ feet}}{t}$. Now t for 100 years is 8400, and the Moon's distance is changed in a century by $\frac{60000}{8400}$ feet or 7·14 feet; in one year it is changed by 0·0714 feet, or less than an inch. This change proceeds uniformly; for every year before the epoch, the distance is additionally greater than the computed distance by the multiple of 0·0714 feet, and for every year after the epoch, it is additionally less.

Wherever Bb occurs, we may use the value found thus. In a century, δv, or ht^2, or $-$ Bb . t^2, or $-$ Bb $\times$ $(8400)^2$, $= 60000$ feet. Therefore Bb (without respect of sign) $= \frac{60000 \text{ feet}}{(8400)^2} = \frac{6 \text{ feet}}{(84)^2}$ $= 0{\cdot}0101$ inch.

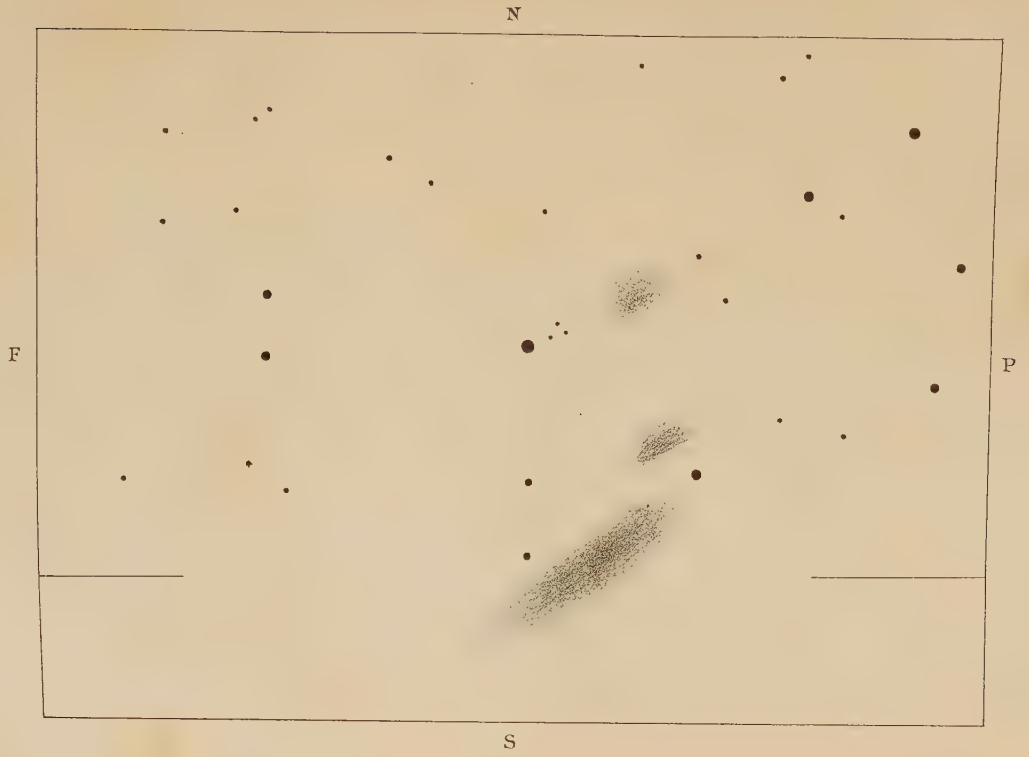
Royal Observatory, Greenwich,
1880, *April* 24.

The Nebula in the Pleiades. By A. A. Common, Esq.

An observation of this Nebula was made on February 8th, 1880, with my three-feet telescope, and the sketch made at this time, a copy of which is attached, differs so from that of Mr. Maxwell Hall in the January number of the *Monthly Notices* (which came to hand soon after) that it may be worth recording. The stars on this sketch are traced from two photographs taken that night with one and a half minute's exposure, and are given as a guide to the positions of the nebulæ seen.

The oval patch of light near *Alcyone* was only seen on this night; later observations made with a view to correct the place of the nebula proper did not show it, but the nights subsequent to the first were not so fine. The smaller patch of light north of *Merope* was always seen; the edge near that star is pretty sharply defined, with dark sky between. The general shape and direction of the nebula proper was as shown. A fine night was waited for with the hope of seeing better the extent of this nebula, but without success. There were pretty certain indications of an extension beyond *Merope* in the direction of *Electra*. In apparent brightness the sharp edge of the smaller nebula was equal to the brighter part of the large one; but this may have been due

NEBULÆ IN THE PLEIADES.



February 8th 1880.
A. A. Common,
Ealing.

Spottiswoode & Co Lith London.

to the contrast the edge made with the sky in the one case, and the gradually brightening towards the middle in the other.

There is a great deal yet to be settled as to the extent and number of the nebulæ in this cluster, and the remarkable difference in the direction given of the principal line of the large nebula is quite incomprehensible.

Rotation Period of Jupiter. By T. D. Brewin, Esq.

In observing Jupiter for the first time last Opposition, August 7th, 1879, $11^h\ 30^m$ G.M.T., I saw a very conspicuous red spot in the southern hemisphere; the preceding end of red spot at the time was as near central as I could judge by eye-estimation. I made a drawing of the planet, thinking it would be a favourable opportunity to obtain the rotation period from sketches taken when the preceding end was central, and at as great an interval of time between successive sketches as possible.

The last sketch of the spot when the preceding end was central I obtained February 4th, 1880, $5^h\ 30^m$ G.M.T., a period of 180 days 18 hours from the first drawing, equal to 437 rotations, the result obtained being $9^h\ 55^m\ 34^s{\cdot}1$, a result so near to Mr. Pratt's described in the January *Notices* of the R.A.S. that I thought it would be of interest to send an account as confirmatory of Mr. H. Pratt's.

14, *St. Nicholas, Leicester*,
1880, *April* 9.

Observations of the Great Southern Comet, 1880, *made at the Melbourne Observatory.* By R. L. J. Ellery, Esq., Director of the Observatory.

The Comet whose appearance I announced by the last mail was observed here from the 9th till the 17th instant. After its first apparition it became rapidly fainter, while the tail increased in length up to 45°; by the 12th, however, the tail had almost disappeared, and on the 17th—the date of our last observations—the nucleus could only be observed with considerable difficulty in a dark field. The moonlight and the Comet's rapid diminution in brightness have now put it out of reach of further measures. A table of apparent places deduced from our observations is annexed.

Observatory, Melbourne,
1880, *February* 20.

Apparent Places of Comet, 1880, from Observations at the Melbourne Observatory.

Melbourne Mean Time.	Comet α	Comet NPD	Observer	Instrument.	No. of Measures.	Comet—Star α	Comet—Star NPD	Name of Star.	Remarks.
d h m s	h m s	° ′ ″				m s	′ ″		
Feb. 9 9 9 30·5	23 41 6·73	123 43 43	W	N	3	+4 1·90	− 1 8·4	Wash. 10462	
9 9 31 44·6	23 41 22·22	123 44 0	W & E	N	3	−4 18·57	− 3 24·0	Wash. 10520	
10 8 40 28·7	23 57 55·76	123 45 8	W	N	3	−2 11·30	+15 59·3	Wash. 10648	
10 9 18 42·6	23 58 22·96	123 44 59	E	S	3	−1 44·10	+15 50·4	Wash. 10648	
14 9 44 46·0			W	N	3	+1 33·67	− 8 2·0	7½ Mag:	about 1° 0′ 23″ RA, 122 29 20 NPD — comp.*
14 9 59 26·7	1 2 5·62	122 21 8	W	N	1	+5 22·55	+ 9 10·5	σ Sculptoris	
15 8 46 4·2	1 15 22·38	121 46 26	W	N	6	−2 34·51	+11 59·9	BAC 421	
16 8 27 17·3	1 28 23·92	121 5 36	E	S	3	+2 13·06	+11 34·4	Wash. 719	Comet excessively faint
16 9 43 23·7	1 29 5·35	121 3 24	W	N	7	+2 54·49	+ 9 22·0	Wash. 719	,,
17 8 57 29·8	1 41 9·51	120 21 43	W	N	6	−1 7·74	+ 1 16·9	Lacaille 522	Comet barely visible; measures little better than guesswork.
17 9 42 2·7	1 41 31·32	120 20 36	E	S	7	−0 45·93	− 0 0·4	Lacaille 522	

In the observer's column E refers to Mr. Ellery, the director, and W to Mr. White, the chief assistant.
In the instrument column S refers to the South Equatoreal of 8 inches' aperture, and N to the North Equatoreal of 4½ inches' aperture.
The observations were made with a rhombus micrometer.

Observations of the Great Southern Comet, 1880, *made at the Observatory, Sydney.* By H. C. Russell, Esq., Director of the Observatory.

I send the following account of a Comet seen here, for insertion in the R. A. S. *Notices* if it is deemed of sufficient importance.

The earliest notice of the Comet which I can obtain is from a gentleman living a '*Moree*' in the northern part of this colony. On Sunday, February 1, he went out shooting quail just after sunset, and on looking to S.W. was surprised by a bright streak of light, stretching from the horizon towards the South Pole. It was a striking object from its brightness, and about three degrees wide (he does not give the length). On Monday he saw it again but it was much fainter; this evening it was seen by many persons in different parts of this and the neighbouring colonies. The nights of February 1 and 2 were cloudy in Sydney, but on the 3rd I saw the comet's tail; between the clouds I could trace it for 30° to a bank of clouds 5° above the horizon. At its widest part, near *β Gruis*, the tail was $1\frac{1}{4}$ degree wide (I enclose sketch taken on 5th). On 4th and 5th the tail was seen; but owing to clouds I could not detect the nucleus. On the 6th it became cloudy and wet until February 13th when I saw the nucleus in (approximately) R. A. $0^h\ 41^m$, Decl. 32° 57′ south. On the 14th another glimpse was obtained, giving approximate position R.A. $1^h\ 1^m\ 9^s$, Decl. 32° 23′ south. In both cases only small stars of comparison could be used, and they will have to be observed with the transit before exact position can be obtained. On the 13th comet had faded so much that it was difficult to see any part of it with the naked eye. The nucleus was very faint, about 5 seconds (time) long, and about 1 minute (arc) wide; the tail at the head was about 30″ wide, about 15′ at its widest part, and about 15° long. On the 14th it turned cloudy again after a few minutes clear sky, and it does not seem probable that the comet will be again observed, as it is fading so rapidly. In the southern colonies the weather has been more favourable.

Sydney Observatory,
1880, *February* 16.

(*From a Circular of the Vienna Academy.*)

COMET 1880 *b* (SCHABERLEON).

Elements :—

Per. Pass. 1880, June 11·7666 Berlin M.T.

Long. Per.	25°	51′	19″	Mean Eq. 1880·0
Long. Node	254	17	15	
Inclination	123	40	13	

Log. Per. Dist. 0·28667.

Computed by Drs. Holetschek and Zelbr, from observations on April 10, 11, and 13.

On the relative Star Magnitude of Mars in February and March, 1880. By Lord Lindsay.

At the request of Mr. Marth, the planet Mars was observed at Dunecht with a Zöllner Astro-photometer on the most favourable nights at the end of February and during March, using *α Tauri* and *α Orionis* as comparison stars. The sky was generally far from favourable, but by watching faint stars in the immediate neighbourhood of the object under observation it is hoped that the final results at least are not seriously influenced by cloud. No readings have been rejected, except two sets specially noted at the time as being affected by haze.

The single results, derived from two readings for each object, are very discordant. This is doubtless owing in part to the very red colour of the planet, which is far more intense than that of either of the stars. The observer, too, found a singular difficulty in avoiding blunders in reading the four quadrants into which the intensity-circle is divided, counting, as they do, in various directions. The very slight arithmetical advantage got by this arrangement is probably far more than negatived by this additional source of error.

The final results for about $8\frac{1}{2}^{h}$. G. M. T., corrected for the varying extinction at different altitudes, are as follows:—

		Observations.	α Tauri ×	α Orionis ×
1880, Feb.	24	♂ = (3)	1·30	...
	26	(2)	1·12	1·23
	29	(1)	1·39	...
March	3	(2)	1·19	1·01
	7	(5)	1·03	1·04
	8	(5)	1·26	1·07
	9	(3)	1·09	0·98
	21	(4)	0·85	0·83

From these it seems that Mars = *α Tauri* March 13·8; and Mars = *α Orionis* March 10·3; or Mars half way between magnitude of *α Tauri* and *α Orionis* at Greenwich noon, March 12, 1880. On the average *α Orionis* = 1·067 × *α Tauri*. All the observations were taken by Ralph Copeland.

Dunecht Observatory,
1880, *April* 6.